IMPACT OF SO₂ EMISSION LIMITS ON PETROLEUM REFINERY OPERATIONS II: MINIMIZING EMISSIONS

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ABSTRACT

The first part of this article presented a general-purpose linear programming model and applied it to an existing petroleum refinery in India to evaluate the impact of imposed maximum SO_2 emission limits on operations and profitability. The present study presents two-step solution methodology designed to minimize SO_2 emission rates while preserving refinery profit. The proposed two-step procedure identifies an alternate solution of the LP model leading to an operating plan with maximized profit and minimized SO_2 emission rates. The study also shows that the alternative of increasing low-sulfur crude processing for lowering the total SO_2 emission rate may be effective only up to a certain proportion of the low-sulfur crude.

INTRODUCTION

Refineries emit SO₂ from their process units and captive power plant stacks. Maximum permissible limits of total SO₂ emission rates from the refinery are set by pollution control authorities with due consideration to local environment and ecology [1-4]. Linear Programming (LP) models are accepted modeling techniques for refineries [5-7]. Refinery LP models are formulated for maximization of profit, typically over a period of one month. A general-purpose LP model has been

developed in the first part of this article to estimate the impact of SO_2 emission limit on refinery profit and operation [8]. The solution of the LP model gives an optimum operating plan for that period, specifying the crude and secondary unit throughputs, routing of streams from process units to different product blends and secondary unit feed blends, all product properties, and product quantities. Operation of a typical fuel oil block of an existing Indian refinery over a month was also modeled using developed LP model.

The maximum SO₂ emission limit for the refinery is met by the design configuration and operation of the refinery facility, while processing the design crude quality. However, in actual day-to-day operation, the design crude mix is rarely available. The available capacities of different processing units of the refinery for the plan period are also often different from the design values over the period of planning due to scheduled shutdowns for plant turnaround and equipment inspection. The optimal planning of the refinery operation for these cases are found using the LP model of the refinery. The optimized operating plan based on the LP solution must respect the maximum SO₂ emission limit fixed for the refinery that may constrain the profit in many cases. The best operating plan, therefore, should maximize profit as well as minimize the SO₂ emission. A methodology for obtaining this 'best' alternate solution is explored. The methodology should lead to a complete operating plan of the refinery, information specifying all process plant throughputs and blends which are required for implementing the plan. It is possible to find only the minimum SO₂ emission value for the maximized refinery profit obtained in the LP solution by using the reduced costs, dual prices, and other information from the sensitivity analysis of the problem around the solution point [9, 10]. However, this procedure for deriving the minimized SO₂ operating plan with maximized refinery profit is not suitable, as it would not spell out a complete operating plan for the refinery.

Therefore, in the present study a two-step LP based methodology is proposed to derive such an operating plan. In the third part of this article, uncertainties in profit and SO_2 emission estimates predicted by model will be presented and discussed.

PROPOSED TWO-STEP METHODOLOGY

In the first step, the LP model with the objective function of maximizing the profit is solved with a maximum SO_2 emission limit constraint. The maximized profit is noted from the solution. In second step, the model is set up with an objective function of total SO_2 emission rate from the refinery to be minimized with an equality constraint on refinery profit set equal to the maximized profit value obtained in the first step. The solution of this LP problem represents a plan of refinery operation at maximized refinery profit and minimized level of SO_2 emission. The minimized SO_2 emission (objective function value) is noted from the solution.

The proposed methodology is general and may be applied to any refinery configuration. However, this article will illustrate above-mentioned methodology with the aid of several case studies for the fuel refinery scheme already discussed in the first part of this article.

REFINING SCHEME

The complete details of the configuration of an existing petroleum refinery in India have been provided in the first part of this article. However, for the reader's convenience, the salient features are outlined. The refinery configuration consists of two Crude distillation units (CDU-I and CDU-II), Visbreaking unit (VBU), Kero-hydro-desulfurization unit (KHDS), Catalytic reforming unit with its feed pre-treater (CRU) and a Vacuum distillation unit (VDU). The refinery is equipped with a Sulfur recovery unit (SRU) to convert H₂S in fuel gas (FG) to elemental sulfur. It can process either or both high sulfur (cheaper) crude and low sulfur (costlier) crude. Streams produced from each unit are shown schematically in Figure 1. Destinations of these streams to different process unit feed and fuel gas and refinery fuel oil pools are shown in Figure 2. Figure 3 shows the streams blended to make various products, viz. liquefied petroleum gases (LP), straight run naphtha (NP), motor spirit (MS) (gasoline), aviation turbine fuel (AF), kerosene (SK), high speed diesel (DL), jute batching oil-grade C (JC), jute batching oil-grade P (JP), fuel oil (FO), lube oil base (LOBS), and stock raw cuts: spindle oil (SO), light oil (LO), intermediate oil (IO), heavy oil (HO), and short residue (SR).

Refinery fuel gas pooled from different process units is washed with amine solution to remove the accompanying H_2S and the sweetened fuel gas (free of H_2S) is consumed in different furnaces of the process units. The amine solution stream picking up the H₂S from the fuel gas is regenerated. H₂S recovered from the regeneration system is fed to SRU and 94 percent [1] of it is converted to elemental sulfur and the rest is emitted as SO₂ from the incinerator stacks. Some specific heavier liquid hydrocarbon streams, such as reduced crude oil (RCO) and short residue (SR), are blended to meet the liquid fuel requirement of the furnaces in the process units and the captive power plant of the refinery. This stream is called the refinery fuel oil (RFO). The SO₂ emission from the refinery is due to the unrecovered sulfur in the SRU stack and burning of the RFO, which contains sulfur compounds, in the different furnaces and captive power plant boilers of the refinery.

GENERATING REFINERY OPERATING PLANS WITH MAXIMIZED PROFIT AND MINIMIZED SO₂ EMISSION

It was observed in part I that the refinery, while operating with maximized profit, might operate at different SO₂ emission rates. This signifies the existence of

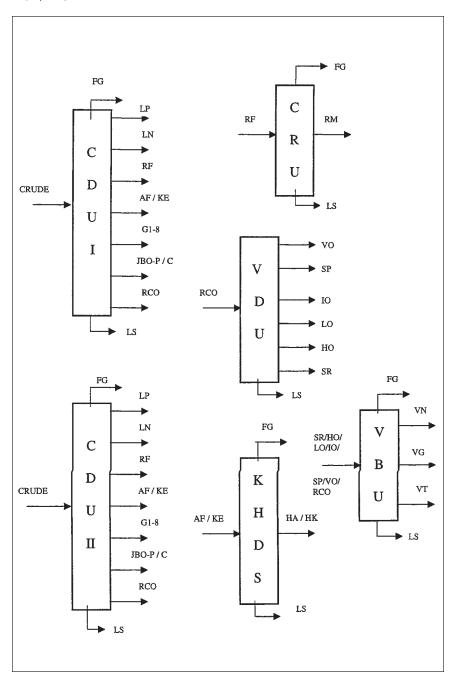


Figure 1. Streams produced in different process units.

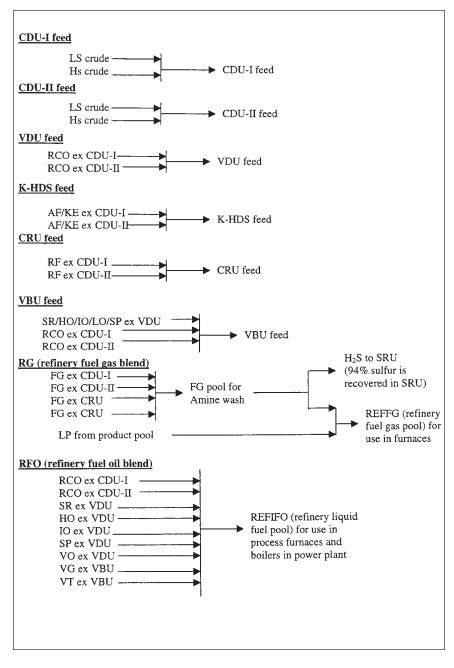


Figure 2. Destination of streams to process unit feed and fuel blend pools.

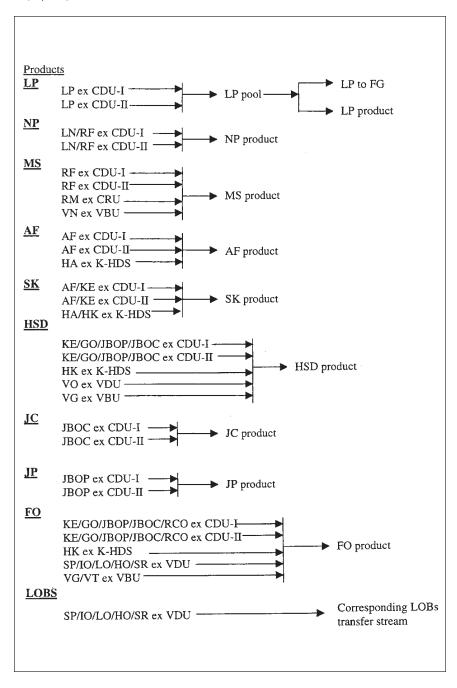


Figure 3. Destination of streams to product blends.

alternate "economically optimum" operating plans with minimized SO₂ emissions from the refinery, through alterations in process unit operations and blending that might result in "source reduction" of SO₂.

Case of Free Crude Mix

As discussed in part I, if the modeled refinery processes only high-sulfur crude and maximizes its profit, the estimated SO₂ emissions from the modeled "fuel refinery" processing is 1207 kg/hr. Since the original design of the refinery is for processing only high-sulfur crude, a figure of 1200 kg/hr is taken as the limit on SO₂ emissions. A typical operating plan and a corresponding SO₂-emissionminimized plan were generated following the two-step methodology, with the refinery free to choose the mix of LS and HS crude. The first plan has a (maximized) refinery profit of Rs 295.8 million/month at SO₂ emission levels of 1180 kg/hr. The corresponding SO₂-emission-minimized operating plan for the refinery shows 985 kg/hr SO₂ emissions for the same profit (16.5 percent SO₂ reduction). The main features of the two operating plans are presented in Table 1. A comparison of the operating plans shows that the reduction in SO₂ emission is due to blend rearrangements between VDU feed, VBU feed, and RFO. The rearrangement results in lower percent sulfur in the RFO blend in the total SO₂-emission-minimized plan. It is interesting to note that even in the presence of a choice to process low-sulfur crude, no low-sulfur crude is chosen due to its higher cost, and the maximum SO₂ emission limit remains high enough yet does not constrain refinery profits.

Case of Fixed Crude Mix

Due to limited availability of crude oil, the refinery often has to plan for processing a fixed crude mix. The plan has to respect the applicable maximum SO₂ emission limit. As a typical example of such a case, a crude mix of 95 percent HS and 5 percent LS crude is considered. The applicable maximum SO₂ emission limit is arbitrarily considered to be 695 kg/hr (500 MT/month). The implication of this limit value has been already discussed in part I of the article as a typical value which constrains refinery profit. Such a reduced limit of maximum SO₂ emission may also be concomitant with the idea that maximum emission limits should be reduced to contain the environmental impacts. The corresponding refinery operation plan was developed by running the LP model for maximized profit at the indicated SO₂ emission limit of 695 kg/hr (case 1).

Now following the two-step procedure, a refinery operation plan with the same crude mix (5 percent LS crude) was generated for maximized profit at minimized SO₂ emission limit (case 2). To assess the effect of fixing the crude mix on minimized-SO₂ operation of the refinery, a third plan with free crude mix and the same profit level was generated. In this case, refinery profit was set equal to that obtained in cases 1 and 2 and SO₂ generation was minimized (case 3).

Table 1. Comparison of Refinery Operation Plans for: 1) Maximized Profit and Meeting Maximum SO_2 Limit; and 2) Maximized Profit and Minimized SO_2 Emission Rate

and 2) Maximized Front and Minimized SO ₂ Emission hate						
Description		Maximized profit and meeting maximum SO ₂ limit (1)	Maximized profit and minimized SO ₂ emission rate (2)			
Maximum SO ₂ emissio	n limit (kg/hr)	1200	_			
SO ₂ emission rate (kg/l	hr)	1180	985			
Profit (million Rs./mont	h)	295.8	295.8			
Crude throughput (thousand MT/month	HS LS	450 0	450 0			
%LS crude		0	0			
Throughput (thousand MT/month)	CDU CRU K-HDS VDU VBU	450 14.5 37.0 128.7 29.6	450 14.5 37.0 128.7 29.6			
VDU feed blend (thousand MT/month)	Stream R3 Stream R4 Total	117.2 11.5 128.7	47.9 80.8 128.7			
VBU feed blend (thousand MT/month)	Stream HO Stream R4 Stream SP Stream SR Total	0.15 27.1 0.0 2.5 29.6	0.15 0.0 9.1 20.5 29.6			
RFO blend (thousand MT/month	Stream LO Stream SP Stream SR Total	0.0 0.02 9.24 9.26	4.43 0.0 4.83 9.26			
Refinery fuel (thousand MT/month)	FG RFO	2.45 9.26	2.45 9.26			
SO ₂ emission from (kg/hr)	FG RFO	150.8 1029.2	150.8 834.8			
%S in RFO		4.0	3.23			

The three operating plans are presented in Table 2. It is seen that the reduction in SO₂ emission in case 2 compared to case 1 is due to lower percent sulfur in the RFO. This is achieved only through blend composition re-adjustments, as can be seen from the table. Similar observations were also recorded earlier with free crude mix cases. A comparison of the SO₂-minimized plans with fixed crude mix (case 2) and free crude mix (case 3) show that at the same profit level, the refinery can reduce its SO₂ emission further (from 568 to 447 kg/hr), if it were free to process a lower proportion (1.7 percent) of LS crude instead of the fixed proportion of 5 percent. This operation plan uses a higher percentage of low-priced HS crude. The secondary units throughputs are lower, and so is the value/profit addition from the secondary units. The loss of profit due is counter-balanced by using a higher proportion of HS crude, so that overall refinery profit remains the same. The lower secondary activities consume lower amounts of RFO and generate less SO₂. Also, the low-sulfur components in RFO increase in proportion, lowering the percentage sulfur in RFO that is also responsible for reduced SO₂ emissions. However, it may be noted that for the crude mix in case 3, the profit is not maximized. A higher profit would result in a corresponding increase in total SO₂ emissions from the refinery.

The above discussion indicates that for the same profit level, increased LS processing may not necessarily reduce the total emission of SO_2 from the refinery. However, "increased LS crude processing generates low-sulfur RFO components and reduces SO₂ emission" is the common assumption of the refiners. This myth is further investigated in the next section.

Cases of Different Crude Throughput Levels

A refinery is often forced to operate with the available crude slate that sets the crude mix as well as the crude processing level of the refinery. Here, the minimization of SO₂ emission at different proportions of LS crude mix is studied for different crude throughput levels.

At 100 Percent Crude Throughput

The minimum and the maximum SO₂ emission operating plans with profit maximization were generated following the two-step methodology. The maximized profit unconstrained by total SO₂ emission limit and its corresponding minimum SO₂ emissions were generated by repeating the two-step procedure described earlier, with the LS crude in the crude mix varying from 0 to 100 percent. At each crude mix, the maximum SO_2 emission from the refinery at the maximized profit is also determined by setting the LP formulation, as in step 2, as a maximization problem. The resulting variations in minimized and maximized total SO₂ emission at maximized profit are shown in Figure 4. The profit shows a monotonic decline with increase in the percentage of LS crude in crude mix, as LS crude costs more than the HS crude. For instance, processing around 5 percent LS

Table 2. Comparison of Refinery Operation Plans for: 1) Maximized Profit, Meeting Maximum SO_2 Limit, 5 Percent Crude; and 2) Maximized Profit, Minimized SO_2 Emission Rate, 5 Percent Crude; and 3) Same Profit as in 1 and 2, Minimized SO_2 Emission Rate, Free Crude Mix

and by barner roll		Case – 1	Case – 2	Case – 3
Description			(Maximized profit,	(Same profit as
Maximum SO ₂ emission limit (kg/hr)		695	_	_
SO ₂ emission rate (kg/hr)		695	568	447
Profit (million Rs./month)		283.7	283.7	283.7
Crude throughput HS (thousand MT/month) LS		427.5 22.5	427.5 22.5	442.3 7.7
%LS crude		5.0	5.0	1.7
Throughput (thousand MT/ month)	CDU CRU K-HDS VDU VBU	450 14.5 37.0 128.7 29.6	450 14.5 37.0 128.7 29.6	450 8.7 32.1 115.5 29.6
VDU feed blend (thousand MT/ month)	Stream R3 Stream R4 Total	38.3 90.4 128.7	38.2 90.5 128.7	95.0 20.5 115.5
VBU feed blend (thousand MT/ month)	Stream HO Stream LO Stream SP Stream SR Total	0.0 0.0 9.2 20.4 29.6	0.2 7.6 2.9 18.9 29.6	0.0 0.0 9.2 20.4 29.6
RFO blend (thousand MT/ month	Stream HO Stream LO Stream R1 Stream SR Stream VO Total	0.2 1.7 5.86 1.5 0.0 9.26	0.0 0.0 8.76 0.5 0.0 9.26	0.0 0.0 3.4 0.0 4.37 7.77
Fuel Gas (RG) blend (thousand MT/month)		2.45 0.0 2.45	2.45 0.0 2.45	1.86 0.73 2.59
Refinery fuel (thousand MT/month)	FG RFO	2.45 9.26	2.45 9.26	2.59 7.77
SO ₂ emission from (kg/hr)	FG RFO	150.0 545.0	150.0 418.0	111 336
%S in RFO		2.11	1.62	1.56

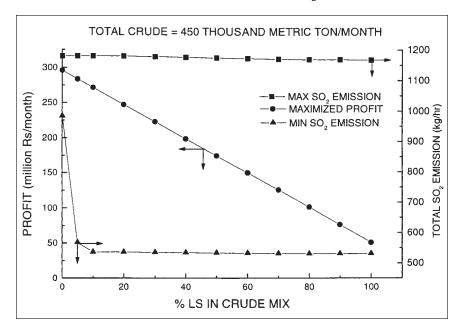


Figure 4. Variations of maximized profit and corresponding minimum and maximum total SO₂ emission with the proportion of low sulfur crude in the crude mix at 100 percent crude throughput.

crude can cut down the minimized total SO₂ emission rate from 985 to 568 kg/hr (42 percent) while the profit falls from 295.8 million Rs./month to 283.7 million Rs./month (4 percent). The minimum possible SO₂ emission level falls sharply up to about 10 percent LS crude in the crude mix, and remains steady thereafter. The LS crude processing generates low-sulfur stocks utilized in making low-sulfur RFO blend. At 100 percent crude throughput level, since the entire demand for this LS stream for RFO can be met with only around 10 percent LS crude, increasing the proportion of LS crude beyond this level does not reduce minimum SO₂ emission any further. Using LS crude above the 10 percent limit leads to spill-over of the low-sulfur components from RFO blend. The maximum SO₂ emission remains more or less constant around 1180 kg/hr.

Thus, processing more than 10 percent LS crude, while the refinery operates at its full throughput, may not be advisable as this cannot decrease the SO₂ emission level but will definitely lower the profit level of the refinery.

Investigations of the effect of maximum SO₂ emission limits on the refinery profit at different crude mix compositions (0, 5, 10, 20, and 30 percent LS crude) were also conducted at the 100 percent throughput level. The results were generated with LS and HS crude processing quantities as equality constraints. Other constraints were kept the same, and the objective (function) was profit maximization. The results, presented in Figure 5, show the effect of maximum SO_2 emission limits on profit for different crude mix at 100 percent crude throughput level. In all cases of different crude mix with above 5 percent LS crude, the profit remains fairly steady when the maximum SO_2 emission limit is above 555 kg/hr. Lowering the maximum emission limit reduces the profit almost at the same rate in all cases. The lowering of profit is due to blend composition changes and restricted throughput of secondary units to reduce fuel consumption and meet lower SO_2 emission limits.

At 60 Percent Crude Throughput

At the minimum turndown level of the refinery, the minimum- and maximum- SO_2 -emission operating plans with profit maximization were generated. The maximized profit and its corresponding minimum and maximum SO_2 emissions were generated for varying percentages of LS crude from 0 to 100 percent. The variations in SO_2 emissions and profits are shown in Figure 6. The profit declines monotonically with increasing percentage of LS crude mix as in the case of 100 percent throughput. The minimum SO_2 emission level is almost steady around

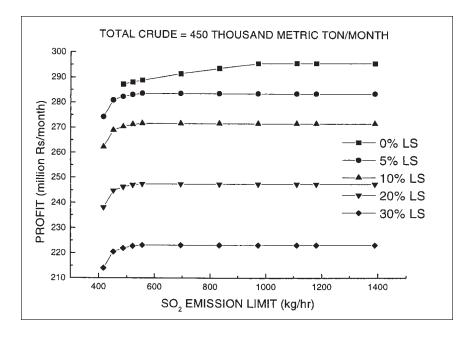


Figure 5. Variations of profit at different crude mix composition with the maximum SO_2 emission limit at 100 percent crude throughput.

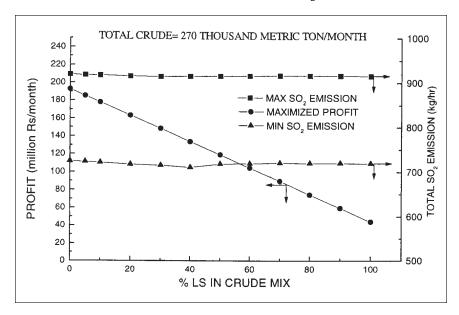


Figure 6. Variations of maximized profit and corresponding minimum and maximum total SO₂ emission with the varying proportion of Is crude in the crude mix at 60 percent crude throughput.

720 kg/hr. At the lower throughput level, the requirement of fuel is less, and can be met from low-sulfur vacuum distillates (VO/SP) and VG from the visbreaking unit. Also, the production of RCO is low at the minimum throughput level, and is almost fully consumed by the VDU to produce profitable product for maximization of refinery profit. So the increase of low sulfur crude processing, which supplies low percent sulfur RCO for RFO pool, does not have much effect on minimizing SO₂ emission at maximized profit.

Maximum SO₂ emissions for different LS crude mixes for maximized profit are almost constant at around 915 kg/hr. This is due to the abundant availability of high-percent-sulfur fuel oil for a smaller volume of RFO pool at this lower throughput level.

At minimum throughput level, due to low RFO requirement, the use of LS crude for the reduction of SO₂ emission is not at all justifiable. The SO₂ emission remains more or less constant, and the profit decreases with the increased use of higher-priced LS crude.

At 60 percent crude throughput level (270 thousand MT/month), the effect of maximum SO₂ emission limits on refinery profit for different crude mixes is shown in Figure 7. At turndown level (60 percent throughput), the profit of the refinery is at its minimum. The variation of the minimum-SO₂ emission constraint

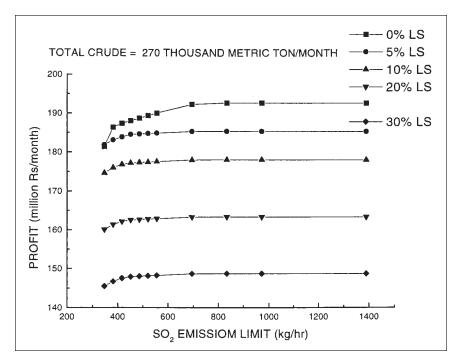


Figure 7. Variations of profit at different crude mix composition with the maximum SO₂ emission limit at 60 percent crude throughput.

does not have much effect on the minimum profit level in all cases of different crude mix. At 5 percent LS crude mix, the profit variation is \sim 3.5 percent, while the SO₂ emission limit comes down from its free level (1380 kg/hr, the maximum) to 332 kg/hr (the lowest).

Minimized SO₂ Emission at Maximized Profit Operating Plans at Different Throughput Levels and Crude Mixes

As refineries often operate at lower (crude) throughput levels due to limited crude availability and/or plant operational problems, the minimized SO₂ emission operating plans for different throughput levels and crude mixes were generated for maximized profit in each case. The LP model was run for 0 percent, 5 percent, 10 percent, 100 percent LS at crude throughput levels ranging from minimum (60 percent) to maximum (100 percent). The maximized profit and corresponding minimized SO₂ emission at various percentages of crude throughput levels of the refinery are shown in Figures 8 and 9, respectively. The profit plot shows that as the crude throughput is increased, the profit also increases. This is true in the case

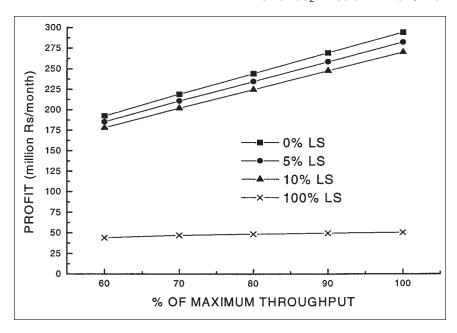


Figure 8. Variations of maximized refinery profit different levels of crude throughput.

of pure LS as well as pure HS crude processing. The rate of increase of profit is sharper in the case of HS crude processing. For pure HS crude to 10 percent LS crude mix processing, the average rate of increase of profit varies from ~2.6 to 2.34 million Rs./month/percent increase of throughput, whereas the same for pure LS crude processing is ~0.17 million Rs./month/percent increase of throughput. The operating costs of units and different product prices are the same for HS and LS crude, but due to the much lower purchase cost of HS crude, the marginal profit is substantially higher for HS crude processing.

The minimum total SO₂ emission plot shows that for pure HS crude processing, as the percent throughput increases, the secondary processing unit throughputs increase so as to maximize profit. Increased secondary throughput is accompanied by higher fuel consumption, resulting in higher SO₂ emissions. This makes the level of minimum total SO₂ emissions increase with the increase in the crude throughput level of the refinery. In the case of pure LS crude processing in the range of 70 percent crude throughput level and higher, the increase in the secondary units' processing leads to higher fuel consumption and higher levels of minimum total SO₂ emissions from the refinery. At a 60 percent throughput level of pure LS crude, the available RCO quantity is lower than in the 70 percent throughput case. The secondary unit throughputs are also consequently lower

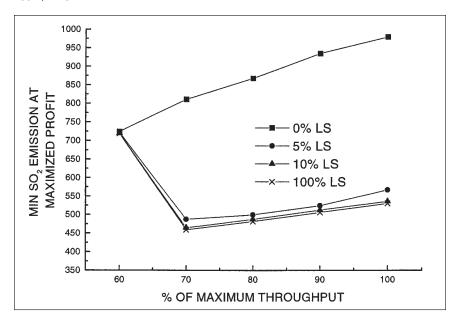


Figure 9. Variations of minimum total SO₂ emission at maximized profit different levels of crude throughput.

than in the 70 percent throughput case. The quantity of RCO is consumed mostly as VDU feed to produce the raw vacuum cuts (SO/LO/IO/HO/SR) for sale as products, and the LS crude RCO for RFO blending is lower compared to the 70 percent crude throughput case. Therefore, for minimized SO $_2$ emission at maximized profit, the RFO blend at the 60 percent throughput level contains a higher proportion of SR and has higher percent sulfur. This causes the minimum total SO $_2$ emission at maximized profit at the 60 percent crude throughput level to rise above that in the 70 percent crude throughput case, as SO $_2$ emission is not a limit for the maximum profit of the refinery. The lower fuel consumption at lower throughput level is not enough to counterbalance the increased emission due to higher-percent sulfur in the RFO blend. This shows that depending on the product demand, the crude nature, and unit capacities, the minimum total SO $_2$ emission from a refinery for maximum profit need not always monotonically increase with increase in crude throughput level.

CONCLUSIONS

Alternative solution(s) may exist for a refinery LP model. The operating plan with minimum SO₂ emission from the refinery with the maximized profit is the best acceptable alternative solution as achieved through the two-step procedure

suggested in this work. In this study, the attempts of the refinery to alter its blending operations to reduce the percent sulfur in RFO as the first stage of reduction of total SO₂ emission from the refinery has been simulated. It is found that the use of more than a particular proportion of low-sulfur crude does not necessarily decrease the SO₂ emission level. In the modeled refinery, processing around 5 percent LS crude at full throughput reduces profit by around 4 percent, while the SO₂ reduction is around 42 percent.

At minimum turndown level (60 percent crude throughput), increasing the percentage of LS crude for the reduction of SO₂ emission is not very effective. The profit increases with the increase in throughput for all crude mixes, but the incremental profit is much higher for HS crude processing due to its lower cost. The minimum SO₂ emissions from a refinery for maximum profit need not always monotonically increase with the crude throughput level. It is clear that the proposed two-step LP model can be used for on-line optimization of refinery operations subject to the prevailing product demand, crude nature, and refinery configuration. In the third part of this article, the uncertainties in profit and SO₂ emission estimates predicted by model will be presented and discussed.

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